Statistical formulae and tables

1997 Edition,

Ian C. McKay,

University of Glasgow

Statistical formulae and tables

1997 Edition

These formulae are intended as a concise *aide-mémoire* for students and scientists who have already studied the most common statistical methods, and for use in examinations where quick access to information is important.

The tables have been calculated from first principles and cover a wider range of values than the tables in most textbooks.

Ian C. McKay,
University of Glasgow,
Department of Immunology,
Western Infirmary,
Glasgow G11 6NT
Tel. 0141-211 2591
Fax 0870-130 5222 or 0141-337 3217
Email: Ian@statistics.org.uk

List of statistical formulae

Definition of the symbols S_{xx}, S_{yy}, S_{xy}, $S_0{}^2$ used below:

$$S_{xx} = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2 \qquad S_{yy} = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2$$

$$S_{xy} = \sum xy - \frac{1}{n}\left(\sum x \sum y\right) \qquad S_0{}^2 = \frac{S_{yy} - \dfrac{\left(S_{xy}\right)^2}{S_{xx}}}{n-2}$$

where x and y are observed variables, there being n values of each.

Descriptive statistics

Mean: $\bar{x} = \dfrac{\sum x}{n}$

Standard deviation: $s_x = \sqrt{\dfrac{S_{xx}}{n-1}}$

Variance: $s_x{}^2$

Standard Error of $\bar{x}$**:** $SE(\bar{x}) = \dfrac{s_x}{\sqrt{n}}$

Standard error of a Poisson-distributed variable: If a poisson-distributed variable (such as blood cell count, viral plaque count, no. of nuclear disintegrations in a minute) has a mean value m, the probability that a particular observation will be exactly equal to r is $\dfrac{m^r e^{-m}}{r!}$, and the standard error of a single observation is $\sqrt{m}$.

Standard error of a proportion: If, in a random sample of n individuals drawn from a large population, a proportion p (comprising at least 5 individuals) fall into a particular category (e.g. atopic, blue eyes, voted Liberal etc.) and a proportion q (comprising at least 5 individuals) do not, then p and q each have a standard error approximately equal to $\sqrt{\dfrac{pq}{n}}$. The same formula can be used whether p and q represent proportions, percentages or actual numbers of individuals.

One-Sample t-test: $t = \dfrac{\bar{x} - m}{SE(\bar{x})}$, where µ is the hypothetical population mean (e.g. zero) and there are (n-1) degrees of freedom. If the observed variable x is the difference between matched pairs of data, the test is called the paired-sample t-test.

Two-Sample t-test: $t = \dfrac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\dfrac{1}{n} + \dfrac{1}{m}\right)}}$ where x, y are individual observations in two groups of size n, m and with sample means $\bar{x}, \bar{y}$. S^2 is the pooled variance given by

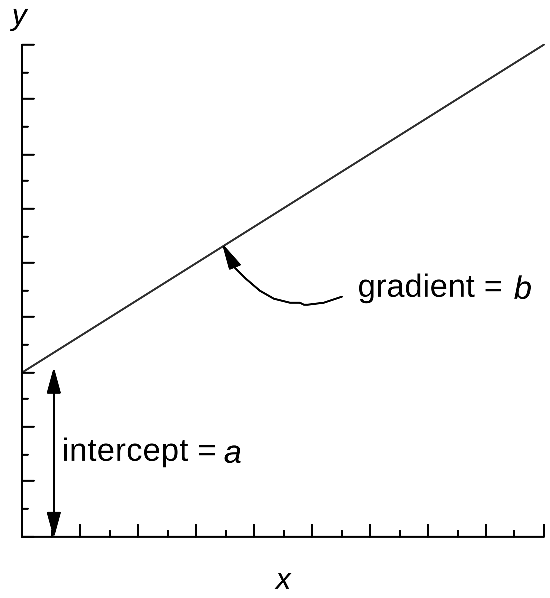
$$S^2 = \frac{(n-1)S_x{}^2 + (m-1)S_y{}^2}{n+m-2},$$ with (n + m - 2) degrees of freedom.

Correlation Coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.

Given the null hypothesis that there is no correlation in the parent population from which the sample was drawn, the test statistic t calculated from the formula $t = r\sqrt{\dfrac{n-2}{1-r^2}}$ has a Student's t-distribution with (n- 2) degrees of freedom, where n = number of subjects observed.

Linear regression of *y* on *x*: The best straight line for predicting y from x is $y = a + bx$, where the gradient is given by $b = \dfrac{S_{xy}}{S_{xx}}$ and the intercept a on the y-axis is given by $a = \bar{y} - b\bar{x}$. The standard errors of a and b respectively are

$$SE(a) = S_0\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \text{ and } SE(b) = \sqrt{\frac{S_0{}^2}{S_{xx}}}.$$



One-way Analysis of Variance: if we have k groups, comprising a total of N observations, we can use one-way analysis of variance to test the null hypothesis that the k populations from which the samples were drawn all have the same mean. The test statistic is $F = \frac{\sum_{i=1}^{k} n_i\left(\bar{x}_i - \bar{x}_G\right)^2}{(k-1)S_{pool}^{\;2}}$, where n_i and $\bar{x}_i$ are the size and mean of group i, $\bar{x}_G$ is the grand mean of all N observations, and $S_{pool}^{\;2} = \frac{\sum_{i=1}^{k}\left(n_i - 1\right)S_i^{\;2}}{N-k}$.

Multiple Comparisons of every group with every other group by Scheffé's Method

The test statistic for comparing the mean, $\bar{x}_i$, of group i with the mean, $\bar{x}_j$, of group j when there are a total of N observations in k groups is

$$F = \frac{\left(\bar{x}_i - \bar{x}_j\right)^2}{(k-1)S_{pool}^{\;2}\left[\frac{1}{n_i}+\frac{1}{n_j}\right]}$$

, which is distributed with (k - 1) on (N - k) degrees of freedom.

See the above section on analysis of variance for definition of $S_{pool}^{\;2}$.

Chi-Squared Test: $c^2 = \sum \frac{\left(\text{observed number - expected number}\right)^2}{\text{expected number}}$.

For an n ´ m contingency table, there are usually (n-1)(m-1) degrees of freedom.
For a goodness-of fit test with n strata, there are usually (n-1) degrees of freedom.

Wilcoxon's Rank Sum Test: if two groups of observations are of sizes N_1 and N_2, and have sums of ranks T_1 and T_2, then the null hypothesis that the two parent populations have the same distributions can be tested by comparing T_1 and T_2 with the critical values shown in the table on page 10. If N_1 and N_2 are outwith the scope of the table, we calculate the test statistic z using the formula:

$$z = \frac{\sqrt{3}\left(N_1(N_1 + N_2 + 1) - 2T_1\right)}{\sqrt{N_1 N_2 (N_1 + N_2 + 1)}}$$

If the null hypothesis is true then z should be a standard normal deviate; i.e. its sampling distribution should be approximately normal with mean of zero and standard deviation 1. The significance of z can therefore be judged by consulting a table of standard normal deviates (see page 11). For example, if z is greater than 1.96 or less than -1.96, then the difference between the two groups is probably significant ($P < 0.05$). This normal approximation is usually adequate if $N_1 > 20$ or $N_2 > 20$ and is a reasonably good guide for most values of N_1 and N_2. The test is designed to be particularly sensitive to differences between the medians of the two populations from which the samples are drawn.

Critical values of Student's *t* statistic for two-tailed tests.

Degrees of freedom	*P* value (probability of exceeding the tabulated value of $\lvert t \rvert$, assuming the null hypothesis)					
	0.1	0.05	0.02	0.01	0.002	0.001
1	6.314	12.71	31.82	63.66	318.3	636.6
2	2.920	4.303	6.965	9.925	22.33	31.60
3	2.353	3.182	4.541	5.841	10.21	12.92
4	2.132	2.776	3.747	4.604	7.173	8.610
5	2.015	2.571	3.365	4.032	5.893	6.869
6	1.943	2.447	3.143	3.707	5.208	5.959
7	1.895	2.365	2.998	3.499	4.785	5.408
8	1.860	2.306	2.896	3.355	4.501	5.041
9	1.833	2.262	2.821	3.250	4.297	4.781
10	1.812	2.228	2.764	3.169	4.144	4.587
11	1.796	2.201	2.718	3.106	4.025	4.437
12	1.782	2.179	2.681	3.055	3.930	4.318
13	1.771	2.160	2.650	3.012	3.852	4.221
14	1.761	2.145	2.624	2.977	3.787	4.140
15	1.753	2.131	2.602	2.947	3.733	4.073
16	1.746	2.120	2.583	2.921	3.686	4.015
17	1.740	2.110	2.567	2.898	3.646	3.965
18	1.734	2.101	2.552	2.878	3.610	3.922
19	1.729	2.093	2.539	2.861	3.579	3.883
20	1.725	2.086	2.528	2.845	3.552	3.850
21	1.721	2.080	2.518	2.831	3.527	3.819
22	1.717	2.074	2.508	2.819	3.505	3.792
23	1.714	2.069	2.500	2.807	3.485	3.768
24	1.711	2.064	2.492	2.797	3.467	3.745
25	1.708	2.060	2.485	2.787	3.450	3.725
26	1.706	2.056	2.479	2.779	3.435	3.707
27	1.703	2.052	2.473	2.771	3.421	3.690
28	1.701	2.048	2.467	2.763	3.408	3.674
29	1.699	2.045	2.462	2.756	3.396	3.659
30	1.697	2.042	2.457	2.750	3.385	3.646
35	1.690	2.030	2.438	2.724	3.340	3.591
40	1.684	2.021	2.423	2.704	3.307	3.551
45	1.679	2.014	2.412	2.690	3.281	3.520
50	1.676	2.009	2.403	2.678	3.261	3.496
55	1.673	2.004	2.396	2.668	3.245	3.476
60	1.671	2.000	2.390	2.660	3.232	3.460
65	1.669	1.997	2.385	2.654	3.221	3.447
70	1.667	1.994	2.381	2.648	3.211	3.435
75	1.665	1.992	2.377	2.643	3.203	3.425
80	1.664	1.990	2.374	2.639	3.195	3.416
85	1.663	1.988	2.371	2.635	3.189	3.409
90	1.662	1.987	2.369	2.632	3.183	3.402
95	1.661	1.985	2.366	2.629	3.178	3.396
100	1.660	1.984	2.364	2.626	3.174	3.391
Infinite	1.645	1.960	2.326	2.576	3.090	3.291

Critical values of the Chi-squared statistic

Degrees of freedom	*P* =				
	0.99	0.95	0.05	0.01	0.001
1	0.000157	0.00393	3.841	6.635	10.83
2	0.0201	0.1026	5.991	9.210	13.82
3	0.1148	0.3518	7.815	11.34	16.27
4	0.2971	0.7107	9.488	13.28	18.47
5	0.5543	1.145	11.07	15.09	20.52
6	0.8721	1.635	12.59	16.81	22.46
7	1.239	2.167	14.07	18.48	24.32
8	1.646	2.733	15.51	20.09	26.12
9	2.088	3.325	16.92	21.67	27.88
10	2.558	3.940	18.31	23.21	29.59
11	3.053	4.575	19.68	24.72	31.26
12	3.571	5.226	21.03	26.22	32.91
13	4.107	5.892	22.36	27.69	34.53
14	4.660	6.571	23.68	29.14	36.12
15	5.229	7.261	25.00	30.58	37.70
16	5.812	7.962	26.30	32.00	39.25
17	6.408	8.672	27.59	33.41	40.79
18	7.015	9.39	28.87	34.81	42.31
19	7.633	10.12	30.14	36.19	43.82
20	8.260	10.85	31.41	37.57	45.31
21	8.897	11.59	32.67	38.93	46.80
22	9.542	12.34	33.92	40.29	48.27
23	10.20	13.09	35.17	41.64	49.73
24	10.86	13.85	36.42	42.98	51.18
25	11.52	14.61	37.65	44.31	52.62
26	12.20	15.38	38.89	45.64	54.05
27	12.88	16.15	40.11	46.96	55.48
28	13.56	16.93	41.34	48.28	56.89
29	14.26	17.71	42.56	49.59	58.30
30	14.95	18.49	43.77	50.89	59.70
35	18.51	22.47	49.80	57.34	66.62
40	22.16	26.51	55.76	63.69	73.41
45	25.90	30.61	61.66	69.96	80.08
50	29.71	34.76	67.50	76.15	86.66
55	33.57	38.96	73.31	82.29	93.17
60	37.48	43.19	79.08	88.38	99.62
65	41.44	47.45	84.82	94.42	106.0
70	45.44	51.74	90.53	100.4	112.3
75	49.48	56.05	96.22	106.4	118.6

The columns most commonly used in this table are those headed 0.05 and 0.01. The entries in the body of the table are the values that must be exceeded by the chi-squared statistic in order to satisfy the test criteria $P<0.05$, $P<0.01$ etc.

Critical values of the Correlation Coefficient

No. of points on graph	*DF*	Probability that the correlation in a sample will exceed the tabulated value of |*r*| given the null hypothesis that the population has zero correlation.					
		P=0.1	P=0.05	P=0.02	P=0.01	P=0.002	P=0.001
		Critical values of the Correlation Coefficient *r*					
3	1	0.9877	0.99692	0.999507	0.999877	0.99999	0.9999988
4	2	0.9000	0.9500	0.9800	0.9900	0.9980	0.99900
5	3	0.805	0.878	0.9343	0.9587	0.9859	0.99113
6	4	0.729	0.811	0.882	0.9172	0.9633	0.9741
7	5	0.669	0.755	0.833	0.875	0.9350	0.9509
8	6	0.621	0.707	0.789	0.834	0.9049	0.9249
9	7	0.582	0.666	0.750	0.798	0.875	0.898
10	8	0.549	0.632	0.715	0.765	0.847	0.872
11	9	0.521	0.602	0.685	0.735	0.820	0.847
12	10	0.497	0.576	0.658	0.708	0.795	0.823
13	11	0.476	0.553	0.634	0.684	0.772	0.801
14	12	0.457	0.532	0.612	0.661	0.750	0.780
15	13	0.441	0.514	0.592	0.641	0.730	0.760
16	14	0.426	0.497	0.574	0.623	0.711	0.742
17	15	0.412	0.482	0.558	0.606	0.694	0.725
18	16	0.400	0.468	0.542	0.590	0.678	0.708
20	18	0.378	0.444	0.515	0.561	0.648	0.679
21	19	0.369	0.433	0.503	0.549	0.635	0.665
22	20	0.360	0.423	0.492	0.537	0.622	0.652
23	21	0.352	0.413	0.482	0.526	0.610	0.640
24	22	0.344	0.404	0.472	0.515	0.599	0.629
25	23	0.337	0.396	0.462	0.505	0.588	0.618
26	24	0.330	0.388	0.453	0.496	0.578	0.607
27	25	0.323	0.381	0.445	0.487	0.568	0.597
28	26	0.317	0.374	0.437	0.479	0.559	0.588
30	28	0.306	0.361	0.423	0.463	0.541	0.570
32	30	0.296	0.349	0.409	0.449	0.526	0.554
37	35	0.275	0.325	0.381	0.418	0.492	0.519
42	40	0.257	0.304	0.358	0.393	0.463	0.490
47	45	0.243	0.288	0.338	0.372	0.439	0.465
52	50	0.231	0.273	0.322	0.354	0.419	0.443
57	55	0.220	0.261	0.307	0.339	0.401	0.424
62	60	0.211	0.250	0.295	0.325	0.385	0.408
67	65	0.203	0.240	0.284	0.313	0.371	0.393
72	70	0.195	0.232	0.274	0.302	0.358	0.380
77	75	0.189	0.224	0.265	0.292	0.347	0.368
82	80	0.183	0.217	0.257	0.283	0.336	0.357
87	85	0.178	0.211	0.249	0.275	0.327	0.347
92	90	0.173	0.205	0.242	0.267	0.318	0.338
97	95	0.168	0.200	0.236	0.260	0.310	0.329
102	100	0.164	0.195	0.230	0.254	0.303	0.321

For a correlation coefficient *r* to satisfy the test criterion defined by the value of *P*, the observed value of *r* must exceed the values shown in the body of the table.

5 per cent points for Variance-ratio (*F*) distribution

This table gives the critical *F* values that are exceeded with frequency 0.05, where *F* is the ratio of two variances s_1^2:s_2^2, and s_1 and s_2 have degrees of freedom ν_1 and ν_2 respectively. Thus, for example, if ν_1 = 2 and ν_2 = 4, the critical value that must be exceeded to satisfy the 95% confidence criterion is 6.94.

	ν_1													
	1	2	3	4	5	6	7	8	9	10	12	15	20	30
ν_2														
1	161	199	216	225	230	234	237	239	241	242	244	246	248	250
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.70
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.38
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.31
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.19
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.15
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.11
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.07
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.01
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	1.98
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	1.96
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.94
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.92
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.90
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.88
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.87
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.85
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.74
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55

1 per cent points for Variance-ratio (*F*) distribution

This table gives the critical F values that are exceeded with frequency 0.01, where F is the ratio of two variances $s_1^2 : s_2^2$, and s_1 and s_2 have degrees of freedom ν_1 and ν_2 respectively. Thus, for example, if $\nu_1 = 2$ and $\nu_2 = 4$, the critical value that must be exceeded to satisfy the 99% confidence criterion is 18.0.

	ν_1													
	1	2	3	4	5	6	7	8	9	10	12	15	20	30
ν_2														
1	4050	5000	5400	5620	5760	5860	5930	5980	6020	6060	6110	6160	6210	6260
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.5
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.8
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.38
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.23
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	5.99
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.20
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.65
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.25
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	3.94
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.70
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.51
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.35
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.21
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.10
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.00
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	2.92
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.84
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.78
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.72
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.67
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.62
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.58
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.54
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.50
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.47
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.44
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.41
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.39
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.20
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.03
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.86

Wilcoxon's rank sum test

Critical values of the rank sum for the 95% confidence criterion. The confidence criterion is satisfied if the observed rank sum is equal to one of the two limiting values listed, or lies outside the limits listed.

Size of other group	Size of group whose rank sum has been calculated																							
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2							36	45	55	66	79	92	106	121	137	155	173	192	212	234	256	279	303	328
							52	63	75	88	101	116	132	149	167	185	205	226	248	270	294	319	345	372
3				15	22	29	38	47	58	69	82	95	110	125	142	159	178	197	218	240	262	286	310	336
				30	38	48	58	70	82	96	110	126	142	160	178	198	218	240	262	285	310	335	362	389
4			10	16	23	31	40	49	61	72	85	99	114	130	147	164	183	203	224	246	269	293	317	343
			26	34	43	53	64	77	89	104	119	135	152	170	189	210	231	253	276	300	325	351	379	407
5		6	11	17	24	33	42	52	63	75	89	103	118	134	151	170	189	209	230	253	276	300	325	352
		21	29	38	48	58	70	83	97	112	127	144	162	181	201	221	243	266	290	314	340	367	395	423
6		7	12	18	26	34	44	55	66	79	92	107	122	139	157	175	195	215	237	260	283	308	333	360
		23	32	42	52	64	76	89	104	119	136	153	172	191	211	233	255	279	303	328	355	382	411	440
7		7	13	20	27	36	46	57	69	82	96	111	127	144	162	181	201	222	244	267	291	316	342	369
		26	35	45	57	69	82	96	111	127	144	162	181	201	222	244	267	291	316	342	369	397	426	456
8	3	8	14	21	29	38	49	60	72	85	100	115	131	149	167	187	207	228	251	274	298	324	350	378
	19	28	38	49	61	74	87	102	118	135	152	171	191	211	233	255	279	304	329	356	384	412	442	472
9	3	8	14	22	31	40	51	62	75	89	104	119	136	154	173	192	213	235	258	281	306	332	359	387
	21	31	42	53	65	79	93	109	125	142	160	180	200	221	243	267	291	316	342	370	398	427	457	488
10	3	9	15	23	32	42	53	65	78	92	107	124	141	159	178	198	219	242	265	289	314	340	368	396
	23	33	45	57	70	84	99	115	132	150	169	188	209	231	254	278	303	328	355	383	412	442	472	504
11	3	9	16	25	34	44	55	68	81	96	111	128	145	164	183	204	226	248	272	296	322	349	376	405
	25	36	48	60	74	89	105	121	139	157	177	197	219	241	265	289	314	341	368	397	426	456	488	520
12	4	10	17	26	35	46	58	71	84	99	115	132	150	169	189	210	232	255	279	304	330	357	385	414
	26	38	51	64	79	94	110	127	146	165	185	206	228	251	275	300	326	353	381	410	440	471	503	536
13	4	10	18	27	37	48	60	73	88	103	119	136	155	174	195	216	239	262	286	312	338	365	394	423
	28	41	54	68	83	99	116	134	152	172	193	215	237	261	285	311	337	365	394	423	454	486	518	552
14	4	11	19	28	38	50	62	76	91	106	123	141	160	179	200	222	245	269	293	319	346	374	403	433
	30	43	57	72	88	104	122	140	159	180	201	223	246	271	296	322	349	377	407	437	468	500	533	567
15	4	11	20	29	40	52	65	79	94	110	127	145	164	184	206	228	251	275	301	327	354	382	412	442
	32	46	60	76	92	109	127	146	166	187	209	232	256	281	306	333	361	390	419	450	482	515	548	583
16	4	12	21	30	42	54	67	82	97	113	131	150	169	190	211	234	258	282	308	335	362	391	421	451
	34	48	63	80	96	114	133	152	173	195	217	240	265	290	317	344	372	402	432	463	496	529	563	599
17	5	12	21	32	43	56	70	84	100	117	135	154	174	195	217	240	264	289	315	342	370	399	429	461
	35	51	67	83	101	119	138	159	180	202	225	249	274	300	327	355	384	414	445	477	510	544	579	614
18	5	13	22	33	45	58	72	87	103	121	139	159	179	200	223	246	271	296	322	350	378	408	438	470
	37	53	70	87	105	124	144	165	187	209	233	257	283	310	337	366	395	426	458	490	524	558	594	630
19	5	13	23	34	46	60	74	90	107	124	143	163	184	205	228	252	277	303	330	358	387	416	447	479
	39	56	73	91	110	129	150	171	193	217	241	266	292	320	348	377	407	438	470	503	537	573	609	646
20	5	14	24	35	48	62	77	93	110	128	147	167	188	211	234	258	283	310	337	365	395	425	456	489
	41	58	76	95	114	134	155	177	200	224	249	275	302	329	358	388	419	450	483	517	551	587	624	661
21	6	15	25	37	50	64	79	95	113	131	151	172	193	216	240	264	290	317	344	373	403	434	465	498
	42	60	79	98	118	139	161	184	207	232	257	283	311	339	368	399	430	462	496	530	565	601	639	677
22	6	15	26	38	51	66	81	98	116	135	155	176	198	221	245	270	296	324	352	381	411	442	474	508
	44	63	82	102	123	144	167	190	214	239	265	292	320	349	379	410	442	474	508	543	579	616	654	692
23	6	16	27	39	53	68	84	101	119	139	159	180	203	226	251	276	303	330	359	389	419	451	483	517
	46	65	85	106	127	149	172	196	221	246	273	301	329	359	389	421	453	487	521	556	593	630	669	708
24	6	16	27	40	54	70	86	104	123	142	163	185	208	232	257	282	309	337	366	396	427	459	492	527
	48	68	89	110	132	154	178	202	227	254	281	309	338	368	399	432	465	499	534	570	607	645	684	724

Table of Standard Normal Deviates

This table lists the proportion P of observations that deviate from the mean of a normal distribution by more than d standard deviations. Half of this proportion P of observations will be greater than the mean plus d standard deviations, and half will be less than the mean minus d standard deviations.

Standard normal deviate d	Probability P of a more extreme value
0.00000	1.00000
0.12566	0.90000
0.25335	0.80000
0.38532	0.70000
0.52440	0.60000
0.67449	0.50000
0.84162	0.40000
1.03643	0.30000
1.28155	0.20000
1.64485	0.10000
1.95996	0.05000
2.05375	0.04000
2.17009	0.03000
2.32635	0.02000
2.57583	0.01000
2.80703	0.00500
2.96774	0.00300
3.09023	0.00200
3.29053	0.00100
3.48076	0.00050
3.61530	0.00030
3.71902	0.00020
3.89059	0.00010
4.05563	0.00005
4.17347	0.00003
4.26489	0.00002
4.41717	0.00001

For deviates greater than those listed in the table, one may use the rule that the proportion P of observations that are more than d standard deviations away from the mean is given approximately in the form of its logarithm by the formula

$$\log_{10}(P) = -0.217\, d^2 - \log_{10}(d) - 0.098 \quad .$$

Classical statistical tests: what are they for, and when are they valid?

All classical statistical tests are applied after observations or measurements have been made on samples of individuals (or cells or mice) drawn from large populations. The aim is to help decide whether some observed effect occurs only in the samples or also in the populations. We start by proposing a null hypothesis, H_0 , namely that the observed effect does not exist in the populations, only in the samples, and has only arisen by random sampling error.

Each statistical test attempts to measure how far the observed evidence differs from what would be expected if H_0 is true. This size of the discrepancy between prediction and observation is measured by a test statistic (t or r or b or F or χ^2).

From the test statistic, a P value is then calculated, which is the probability of getting a test statistic as big as or bigger than the observed test statistic if the null hypothesis is true. Thus P is the probability of the evidence, given the hypothesis H_0. It is not the probability of the hypothesis given the evidence. If P is very small, the null hypothesis is usually rejected.

Statistical test	**Preconditions and assumptions**	**Purpose of test**
1-sample t	Normally distributed parent population.	To test the hypothesis that a population has a particular mean value, e.g. zero.
2-sample t	Normally distributed parent populations with equal variances.	To test the hypothesis that two populations have the same mean.
Paired t	Differences between pairs are normally distributed in the population.	To test the hypothesis that the mean difference of paired data in a population is zero.
Wilcoxon's rank sum test	See last row of table.	To test the hypothesis that two populations have the same distributions.
Linear regression analysis	y-values observed, not chosen, from a normally distributed parent population with the same variance at all x-values.	To find the best-fitting straight line, and test the hypothesis that the gradient or the intercept in the population is zero.
Linear correlation analysis	x and y observed on a random sample of individuals from a population in which x and y are both normally distributed.	To measure the strength of association between two continuous variables and test the hypothesis that the population has a zero correlation coefficient.
One-way analysis of variance	All populations are normally distributed, with the same variance.	To compare samples from several populations and test the hypothesis that all populations have the same mean.
Scheffé's method of multiple comparisons	All populations are normally distributed, with the same variance.	To compare samples from several populations and see which means differ significantly from which, (and by how much).
Chi-square	The data define (in whole nos.) how many individuals fall into various mutually-exclusive categories. Expected frequencies should all be at least 5.	To test whether the numbers of observations in various categories differ significantly from the numbers (frequencies) expected on the basis of a null hypothesis.
All the above tests.	Items of data are mutually independent and have been randomly sampled from a large homogeneous parent population.	To test whether any apparent contradiction to a null hypothesis could have arisen merely from random sampling error.